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Student's Algebraic Thinking Ability in Solving Mathematical Problems Based Misconceptions

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A B S T R A C T

Mastering algebraic concepts and skills has been one of the demands in the Indonesian curriculum for the past few decades. Teaching algebraic concepts and skills is one of the materials taught from the junior high school to university level. Therefore, it is mandatory for pre service mathematics teacher students to master and even fluently use algebraic concepts in order to develop students' algebraic thinking skills. Thus, this study aims to understand and obtain a description of students' algebraic thinking abilities and misconceptions in solving problems. This research using descriptive qualitative. Analysis was conducted using tests and in-depth interviews with students who had taken linear algebra courses in semester 3 of the Mathematics Education Study Program at private university in NTT. Student's algebraic thinking skills in solving problems were analyzed using Kieran's (2004) indicators consisting of: generational activities, transformational activities, and global meta-level activities. The results of the research show that…. In generational activities, students in the low, medium, and high categories have misconceptions. In transformational activities and global meta-level activities, misconceptions are only carried out by low category students.

1. Introduction

Mastering algebraic concepts and skills is one of the demands in the Indonesian curriculum. Algebra is the study of mathematical symbols and the rules for manipulating these symbols as stated by Herstein (1964), meaning that algebra is a part of mathematics that is used to solve other mathematical problems and also helps solve problems in everyday life because Algebra deals with symbols and the rules for manipulating those symbols. Algebra is a branch of mathematics that uses mathematical statements to describe relationships between things (NCTM,

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2000). One of the main strengths of algebra is as a tool for generalizing and solving various problems (Saputro et al, 2018). Learning algebra should be done to develop students' abilities in: 1) understanding patterns, relations, and functions; 2) represent and analyze mathematical situations using symbols and algebraic procedures; 3) using a mathematical model to express and understand quantitative relations; and 4) analyzing changes in various contexts (NCTM, 2000). So that learning algebra emphasizes thinking activities so that students have a good way of thinking after learning algebra. This ability is called the ability to think algebra.

Algebraic thinking is an ability that emphasizes mathematical thinking activities which consist of several activities. There are 3 activities in algebraic thinking, namely generational activities, transformational activities, and global meta-level activities (Kieran, 2004). Generalization activities include forming expressions and equations. The objects that underlie expressions and equations are unknown variables and factors, such as the equal sign and the notion of an equation's solution. Most of the activities of constructing the meaning of algebraic objects occur in algebraic generational activities.

Transformational activities include collecting like terms, factoring, expanding, replacing, adding and multiplying polynomial expressions, exponentials with polynomials, solving equations, simplifying expressions, working with equivalent expressions and equations, and so on. This activity has a lot to do with changing the form of expressions or equations to maintain equality. While global meta-level activities include activities using algebra as a tool but its use is not limited to algebra. Including problem solving, modeling, paying attention to structure, studying changes, generalizing, analyzing relationships, justifying, proving, and predicting (Sukmawati, 2015; Kieran, 2004).

Students with 3 aspects of algebraic thinking ability will be able to extract information by analyzing the relationship between numbers through information exploration in the form of an image of the problem given or an image made by themselves (Riskon et al, 2019). Therefore, students must understand concepts and relationships beyond mere symbolic manipulation, implying that their studies begin in the early years with the development of algebraic thinking, in which the search for patterns and the formation of conjectures and generalizations in figurative contexts play an important role, including the use of representations. different (Barbosa et al, 2022). Algebraic thinking is a thinking activity that emphasizes the use of algebraic symbols in solving mathematical problems, so that this ability is important for students to have and master (Farida & Hakim, 2021).

The importance of algebraic thinking ability for students is in line with what was conveyed by Kieran (2004) that through the development of algebraic thinking ability, students will understand the focus on relations and representations in solving a problem. So that by thinking algebraically, students can solve mathematical problems that occur in their lives and can carry out several activities such as analysis, presentation, and generalization. This means that having good

algebraic thinking ability shows a better understanding of mathematics. As one of the critical aspects of mathematics, algebra provides many benefits for life. However, the facts on the ground show something different. Algebra, which should be important, creates its own problems for students. Based on the results of the initial research, difficulties and errors were found in learning mathematics which caused the development of student's algebraic thinking abilities to be not optimal. These things happen because students do not use their knowledge of algebraic material or the initial concepts possessed by students are not in accordance with the concepts of scientists and can be caused by the students themselves, teachers, teaching methods, textbooks and so on (Dwirahayu et al, 2018). Concept errors can also be called misconceptions.

A study found student errors in solving math problems in algebraic forms, namely procedural errors and technical/calculation errors in comparisons, errors in writing units, and errors in substituting values from given problems. This error was made on the grounds that mathematics is difficult to understand because it is too abstract, so that students often experience difficulties in learning mathematics and cause errors in solving problems. Another study found that the ability to think algebra is still weak due to learning barriers in understanding algebraic concepts, especially when solving problems of algebraic forms. Various misconceptions that were carried out resulted in errors in solving the questions given and of course also affected learning outcomes. Therefore, the misconceptions that are owned should not be allowed to last long in students (Meilani et al, 2022; Badawi et al, 2016).

Problems in learning algebra were also experienced by Mathematics Education students at a private university in NTT. The results of the preliminary study found that student's algebraic thinking ability is low, one of which was proven by the student learning outcomes in linear algebra courses are low, only 36% of students achieved KKM (76). When explored further, it was found that student's basic understanding related to algebra was still low and there were also misconceptions, students were still lacking in doing analysis so that when given questions that were different from the lecturer's example, students experienced confusion and were unable to solve the problem. Therefore, the misconceptions that students have cannot be left alone and last a long time in students. It is important for teachers to know student’s misconceptions so that it becomes the basis for designing learning that can reduce the occurrence of misconceptions for students in solving algebraic problem.

Based on this description, this study will examine students algebraic thinking ability in solving mathematical problems based on misconceptions through research with the title: "Students Algebraic Thinking Ability in Solving Mathematical Problems Based on Misconceptions."

2. Methodology

This type of research is qualitative descriptive research that aims to describe Student algebraic thinking ability and misconceptions that students make. Students algebraic thinking ability were analyzed using indicators which consisted of generational activities, transformational activities, and global meta-level activities (Kieran, 2004). This research involves 3 mathematics education students at a private university in NTT class of 2021. This subject was selected using a purposive sampling technique. Data was collected by tests and interviews. The instrument of this study included two Essay Questions of Gauss–Jordan elimination set operation questions (Table 1) and an interview guideline. The participant answered the questions in 40 minutes. The test is given to determine students' algebraic thinking skills in solving problems using Gauss-Jordan elimination and misconceptions that students make when solving a given problem, while in-depth interviews were conducted to complete the data after the results of the students written works was analyzed.

Table 1. Gauss-Jordan Elimination Questions

Assessment Material
1. In matrices a-d below, determine whether the matrix is in row echelon form, reduced row echelon form, both, or neither a. $\begin{bmatrix}1&2&0&3&0\\0&0&1&1&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix}$ b. $\begin{bmatrix}1&0&3&1\\0&1&4&2\end{bmatrix}$ c. $\begin{bmatrix}1&-6&5&2\\0&1&3&3\end{bmatrix}$ d. $\begin{bmatrix}1&2&0&3&0\\1&0&3&3&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix}$
2. Mr. Marten is a fruit seller and he Sells packages of fruit consisting of salak, oranges and mangoes. Here are the packages sold by Mr. Marten: 3 kg of salak, 2 kg of oranges and 2 kg of mangoes for 197,000 2 kg of salak, 1 kg of oranges and 2 kg of mangoes for 140,000 2 kg of salak, 3 kg of oranges and 1 kg of mango costs 172,000 Calculate the price per kg of each fruit using Gauss–Jordan elimination!

3. Results and Discussion

Student test results will be analysed by focusing on student misconceptions in each algebraic thinking activity that is carried out. The results of the analysis will be described based on 3 categories, namely high, medium and low categories. Based on the analysis of student test results, several misconceptions were found by students as follows:

Misconception in Generational Activities

Generation activities involve forming expressions and equations that become algebraic objects (Kieran, 2004). This means that at this stage activities are carried out to build the meaning contained in the given algebraic object. Such as: 1) determine equations containing unknowns that represent the problem; 2) general

expressions arising from numerical sequences; and 3) the expression of the rules governing numerical relationships. Based on the analysis of student test results for question number 1, it was found that students in the low category had misconceptions about the definition of row echelon form and reduced row echelon form so that they were unable to solve problem number 1 correctly. As shown in Figure 1.

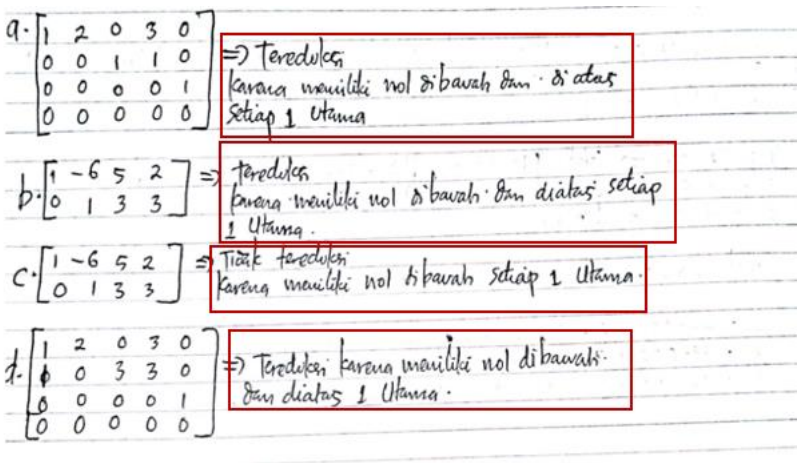

Figure 1. Student Work in The Low Categories

Based on Figure 1, students do not understand the terms of the matrix, it is said to be in row-echelon form or reduced row-echelon form, so the answers given are not in accordance with the same reasons for each question given. Students also do not understand well what is known from the problem and are not careful in analyzing each element of the matrix due to incomplete understanding of students regarding the definition of row echelon and reduced echelon matrices. This is in accordance with research that found the cause of misconceptions is incomplete understanding and their thoughts related to concepts understood by generalizing the lack of understanding concepts (Nurtasari et al, 2017). The reasoning that was carried out was not optimal so that incomplete information was obtained which resulted in wrong conclusions being drawn (Dwirahayu et al, 2018). Misconceptions were also carried out by students in the medium and high categories where it was inaccurate to determine one of the 4 forms of the given matrix. As shown in Figure 2.

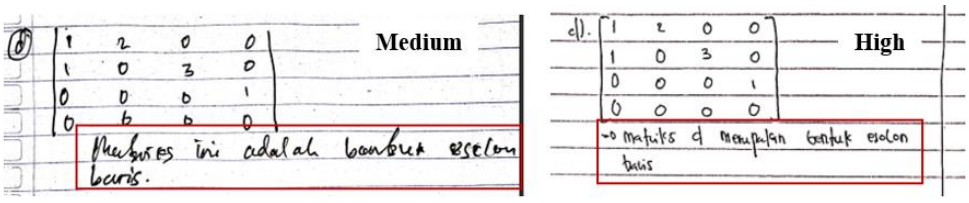

Figure 2. Student Work in The Medium and High Categories

Based on Figure 2, this misconception occurs due to students' incomplete understanding of the conditions that must be fulfilled by a matrix, said to be in row-echelon form or reduced row-echelon form. The student's understanding is

that if the entry of the matrix row starts from 1 then it can be called the row echelon form, but does not pay attention to the next condition where the main 1 entry in the next row must be located to the right of the main 1 entry in the previous row, so the correct answer is the matrix is not is row echelon as well as not reduced row echelon. This misconception occurs due to mistakes and lack of thoroughness by students in analysing the conditions fulfilled by the matrix resulting in inaccurate conclusions on answers. This is in accordance with research that found the misconceptions were made due to erroneous conclusions in concluding concepts that led to misunderstandings in solving the given questions (Nurtasari et al, 2017). In question number 2, students did not make generalization misconceptions from the questions given. Students are able to identify well, known components of the questions given.

Misconception in Transformational Activities

Transformational activities include forming and equating and solving equations from the problems given (Kieran, 2004). This activity also includes student activities in determining the method used and carrying out the method until finding a solution to the problem given. From the questions given in school number 2, it is a question to measure student transformational activities. Based on the results of student work on question number 2, students are able to carry out generalization activities well. Based on the generalization activity, students are also able to transform the problem given in the form of a mathematical model by writing down the known components of the problem correctly. This happens because students are used to changing the form of word problems into algebraic form and are able to determine the right method to solve the problem, namely the Gauss-Jordan elimination method and also the mixed method. Misconceptions made by students in transformational activities, namely in the algebraic form operations carried out by students in the low category are not correct, as shown in Figure 3.

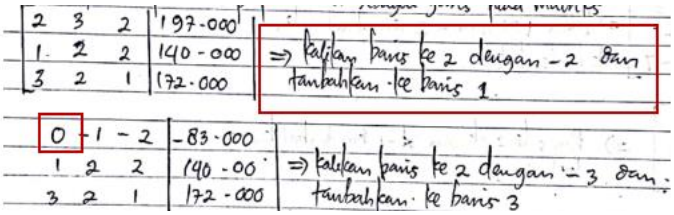

Figure 3. Student Work in The Low Categories

Based on figure 3, students make misconceptions about algebraic operations. The first elementary form operation (OBE) is to determine the main 1 component of the matrix, but students do the OBE and change the first row of the first column of the matrix to zero so this is no longer in accordance with the concept of operations with the Gauss-Jordan method. The misconceptions that were carried out by students in the first step of OBE resulted in the OBE being carried out in the later stages also being wrong and obtaining wrong results. This misconception was carried out by students due to incomplete understanding of students related to OBE in the Gauss-Jordan elimination method so that it was wrong to apply it in

solving the given problem. This is consistent with research that found that in working on transformational activity algebra questions, low category students tend not to be able to perform operations on algebraic forms, and determine the solution of an equation in algebra (Badawi et al, 2016). Students in the medium and high categories do not make transformational misconceptions.

Misconception in Global Meta-Level Activities

Global meta-level activities include activities using algebra as a tool to identify changes, relationships, and predict appropriate methods in mathematics and use them to solve problems related to other fields of science. Measuring the global meta-level activity of students is given question number 2 by paying attention to the way students use to solve these problems using algebraic concepts which are understood by connecting the problem in the form of a system of linear equations so that it uses a mixed method or Gauss-Jordan elimination to determine the price of 1 kg of each unknown fruit. Students with a low category make misconceptions about this activity so that it does not produce the right problem solving, as shown in Figure 4.

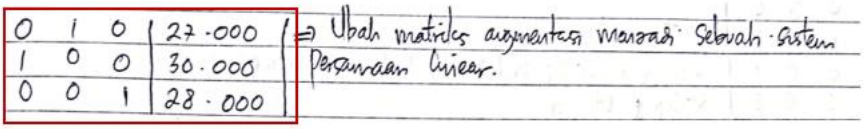

Figure 4. Student Work in The Low Categories

Based on Figure 4, students make misconceptions in analyzing the form of operations used to solve the problems given so that the results of the operations carried out are not in accordance with the concept of Gauss-Jordan elimination where the OBE results form a reduced row echelon matrix, so that from the results of the work, it can be seen that students are doing haphazardly based on an incomplete understanding with important assumptions to produce a matrix that only contains one element with a value of 1 in each row. The results of this work certainly show that students' algebraic thinking skills at global meta-level activities have not been achieved. Meanwhile, students in medium and high categories did not have global meta-level misconceptions as shown in Figure 5.

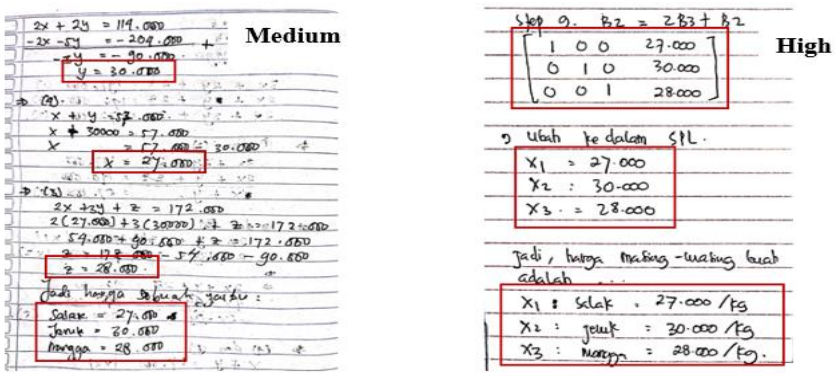

Figure 5. Student Work in The Medium and High Categories

Based on figure 5, students with the medium categories are solving the problem using the elimination-substitution method to determine the value of each unknown variable from the problem. While students in the high categories solve problems using Gauss-Jordan elimination well and obtain the value of each variable that is not known exactly. This is consistent with research that found the medium and high categories are able to carry out global meta-level activities well (Badawi et al, 2016).

4. Conclusion

Based on the results of the research and discussion, it was found that there were misconceptions in students' algebraic thinking activities through solving the problems given. In generational activities, misconceptions are carried out by students in the low category caused by incomplete understanding of students which results in students' thoughts forming inappropriate conclusions. While the misconceptions carried out by students in the medium and high categories were caused by mistakes and lack of thoroughness in analyzing each element in the matrix and the conditions that were met resulting in an erroneous conclusion on one of the question numbers. In transformational activities and global meta-level activities, misconceptions are only made by low category students caused by students' incomplete understanding of elementary row operations and mistakes in applying them so that they cannot solve problems properly.

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